Safety Analysis of Spherical Shell Structures Subjected to External Pressure

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Abstract: To increase the safety of various spherical shell components under pressure environments, the present research deals with the study of elastic -plastic stresses in a spherical shell under the effect of external pressure. The solution of the problem has been obtained by using the Seth's transition theory of elastic-plastic transitions. The transition theory does not assume classical assumptions like incompressibility and yield conditions. The radial and circumferential stresses have been calculated for the spherical shell for compressible as well as incompressible materials. It has been observed that the spherical shell made of incompressible material requires high pressure to start initial yielding in the shell as compared to spherical shell made of compressible material. The results derived are shown graphically.

Keywords: elastic-plastic, pressure, spherical shell, stresses.

Introduction

Analysis and design of spherical shell structures in aerospace, chemical, civil and mechanical industries such as in high-speed centrifugal separators, gas turbines for high-power aircraft engines, spinning satellite structures, certain rotor systems and rotating magnetic shields is important for safety purpose and long life of shell structures [1]. To increase the life of spherical shells subjected to the external pressure, it is therefore very important for engineers to study the safety analysis in the spherical shells under various environments. Zhang *et.al.*[2] discussed buckling behaviors of the spherical shells under uniform pressure. Analyses involved considering the average geometry, average wall thicknesses, and average elastic material properties. Numerical calculations entailed considering the true geometry, average wall thicknesses, and elastic-plastic modeling of true stress–strain curves. Cong *et.al.* [3] discussed the nonlinear axi-symmetric response of shallow spherical FGM shells under mechanical, thermal loads and different boundary conditions based on classical theory of shells. [4]Shell structure with Kratzer confining potential has been theoretically investigated by Hayrapetran in the framework of the effective mass approximation. It is shown that with the increase of the hydrostatic pressure, the diamagnetic susceptibility increases. Viola *et.al.* [5] studied the static behavior of functionally graded spherical shells and panels subjected to uniform loadings at the extreme surfaces. The material properties are graded in the thickness direction according to a four parameter power law. The structural model involves the a posteriori stress and strain recovery procedure. The obtained governing equations are solved by means of the GDQ numerical technique. An extensive numerical investigation is carried out to characterize the effect of material parameters on the stress, strain and displacement profiles along the thickness direction. These authors have analyzed the problems considering the assumptions (i) incompressibility condition (ii) Creep –strain laws like Norton (iii) Yield condition like that of Tresca (iv) Associated flow rule. The necessity of use of these ad-hoc semi-empirical laws in classical theory of elastic-plastic transition is based on approach that the transition is linear phenomenon which is not possible. Therefore, it suggests that at transition behavior, non-linear terms are significant and cannot be ignored. The concept of generalized strain measures is useful to solve the various problems of elastic -plastic transition by solving the non-linear differential equations at the transition points. This concept of generalized strain measures and transition theory has been applied to find elastic-plastic stresses in various problems; for example Thakur *et.al.*[6-8] analyzed elastic-plastic & creep transition in spherical shell, cylinder and disc with various conditions. All these problems based on the recognition of the transition state as separate state necessitates showing the existence of the constitutive equation for that state. In this paper, we shall derive the results for effective pressure required to start initial yielding in the spherical shell. The stresses under pressure in spherical shell are calculated for compressible as well as for incompressible materials. The results obtained are shown graphically.

Formulation of the Mathematical Problem

We consider here a thick-walled spherical shell, whose internal and external radii are *a* and *b* respectively, is subjected to uniform external pressure *p*. It is convenient to use spherical polar coordinates (r, θ, ϕ), where θ the angle *is* made by the radius vector with a fixed axis, and ϕ is the angle measured round this axis. By virtue of the spherical symmetry $\sigma_\theta = \sigma_\phi$ everywhere in the shell, due to spherical symmetry of the structure, the components of displacement in spherical co-ordinates (r, θ, ϕ) are given by $u = r(1-\beta), v = 0, w = 0$ where *u*, *v*, *w* (displacement components); β is position function, depending on *r* = $\sqrt{x^2 + y^2 + z^2}$ only. Generalized components of strain are given by Seth's [9-10]:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right],\ e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right] = e_{\phi\phi}, e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0, \tag{1}$$

where $\beta' = d\beta/dr$.

Stress-Strain Relation: The constitutive equation of stress –strain for isotropic material is given as [11]:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij},\ (i, j = 1, 2, 3) \tag{2}$$

where symbols have their usual meaning.

By using equation (1) in equation (2), the stresses are obtained as:

$$T_{rr} = \frac{\lambda}{n}\left[3-2\beta^n-(r\beta'+\beta)^n\right] + \frac{2\mu}{n}\left[1-(r\beta'+\beta)^n\right],$$

$$T_{\theta\theta} = \frac{\lambda}{n}\left[3-2\beta^n-\left(r\beta'+\beta^n\right)\right] + \frac{2\mu}{n}\left[1-\beta^n\right] = T_{\phi\phi},$$

$$T_{r\theta} = T_{\theta\phi} = T_{\phi r} = 0 \tag{3}$$

Equation of equilibrium: The radial equilibrium of an element of the spherical shell requires:

$$r\frac{dT_{rr}}{dr} - 2\left(T_{\theta\theta} - T_{rr}\right) = 0 \tag{4}$$

where T_{rr} and $T_{\theta\theta}$ are the radial and circumferential stresses. For sufficiently small values of the pressure, the deformation of the shell is purely elastic. The boundary conditions of problem are

$$T_{rr} = 0 \text{ at } r = a$$

$$T_{rr} = -p \text{ at } r = b, \tag{5}$$

Using Equations (3) in Equation (4), we get a non- linear differential equation in β as:

$$P(P+1)^{n-1}\beta\frac{dP}{d\beta} + P(P+1)^n + 2(1-c)P - \frac{2c}{n\beta^n}\left[\left\{1-\beta^n(P+1)^n\right\} - \left(1-\beta^n\right)\right] = 0 \tag{6}$$

where compressibility $c = 2\mu/\lambda + 2\mu$ and $r\beta' = \beta P$ (*P* is function of β and β is function of *r*). The transition points of β in Equation (6) are $P = 0$, $P \to -1$ and $P \to \pm\infty$. Here by, we are only interested in finding plastic stresses corresponding to $P \to \pm\infty$.

Solution of Problem through Principal Stress

In order to calculate elastic-plastic stresses, we define the transition function by taking the principal stress T_{rr} (see, Thakur, Verma [12-17]) at the transition point $P \to \pm\infty$ .The transition function R is given as:

$$R = (3\lambda + 2\mu) - nT_{rr} = \frac{2\mu\beta^n}{c}\left[(P+1)^n + 2(1-c)\right] \tag{7}$$

Taking the logarithmic differentiating of eq. (7) with respect to *r and* substituting the value of $dP/d\beta$ from eq. (6) and taking asymptotic value $P \to \pm\infty$, after integration we get:

$$R = A_1 r^{-2c} \tag{8}$$

where A_1 is constant of integration.

By using Equations (7) and (8), we have the transition value T_{rr} is

$$T_{rr} = \frac{1}{n}(A_2 + A_1 r^{-2c}) \tag{9}$$

where $A_2 = (3\lambda + 2\mu)$ is a constant.

By using the boundary conditions, we have

$$A_1 = \frac{-np}{b^{-2c} - a^{-2c}}, A_2 = \frac{npa^{-2c}}{b^{-2c} - a^{-2c}} \tag{10}$$

Substuiting the values of the constants A_1, A_2 in equation (9)

$$T_{rr} = -p\frac{(r/a)^{-2c} - 1}{(b/a)^{-2c} - 1} \tag{11}$$

By using the equation (11) in equation (4)

$$T_{\theta\theta} = T_{rr} + pc\left\{\frac{(r/a)^{-2c} - 1}{(b/a)^{-2c} - 1}\right\} \tag{12}$$

Initial Yielding

It is clear from equation (12) that the value of $\left|T_{\theta\theta} - T_{rr}\right|$ is maximum at *r* = *b*. Therefore, yielding of the spherical shell take place at the external surface

$$\left|T_{\theta\theta} - T_{rr}\right|_{r=b} = \left|\frac{pc(b/a)^{-2c}}{(b/a)^{-2c} - 1}\right| \equiv Y \tag{13}$$

Therefore, External pressure required for initial yielding at the external surface is given as

$$P_{external} = \frac{p}{Y} = \frac{1 - (a/b)^{2c}}{c} \tag{14}$$

Fully-Plastic state

For fully plastic state, we make c→0 in equations (14) and we get the following equations

$$\left|T_{\theta\theta} - T_{rr}\right|_{r=b} = \left|\frac{(p_1 - p_2)}{2\log(b/a)}\right| \equiv Y_1 \tag{15}$$

Therefore, Effective pressure and stresses for fully plastic state at the external surface is given as

$$P_f = \frac{p}{Y_1} = 2\log(a/b) \tag{16}$$

$$T_{rf} = -p\frac{\log\frac{r}{a}}{\log\frac{b}{a}}$$

$$T_{\theta f} = T_{rf} + \frac{p}{2\log(a/b)} \tag{17}$$

Discussion of Results

From the above analysis of initial yielding and fully plastic state, the effective pressure is calculated for the both stages. Curves are plotted between effective pressure along the radii ratio a/b (see Figure 1-2) for the spherical shell made of compressible material as well as incompressible material. It has been observed that the spherical shell made of compressiblility C=0.35 material requires high pressure to start initial yielding in the shell as compared to spherical shell made of compressible material c =0.50, 0.75, 1. Further the effect of the pressure is seen on the radial and circumferential stresses of the spherical shell. The radial and circumferential stresses are plotted against the radii ratio r/a with the compressibility factor c = 0.25, 0.50, 0.75. It is observed from Figure3 that the value of circumferential/radial stress is maximum at the internal surface of shell. Further as we increase the external pressure, the value of stresses also increase and lead more damage to the spherical shell.

Conclusion

From the above results, it can be concluded that spherical shell made up of incompressible material is on the safer side of the design as compared to the spherical shell made up of compressible materials under pressure. The main reason is due to high pressure required for start initial yielding in the spherical shell which leads to the more safety and life of the spherical shell.

Acknowledgement

The corresponding author wishes to acknowledge his sincere thanks to respected Professor D. S. Pathania (Department of Mathematics), Guru Nanak Dev Engineering College, Ludhiana, Punjab for his encouragement during the preparation of this paper.

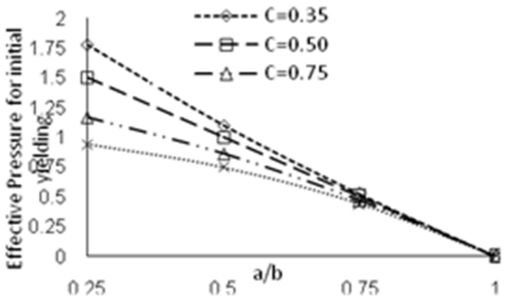

Figure1.Pressure required for initial yielding

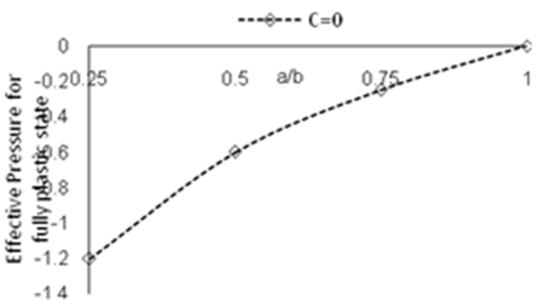

Figure2. Pressure required for fully plastic state

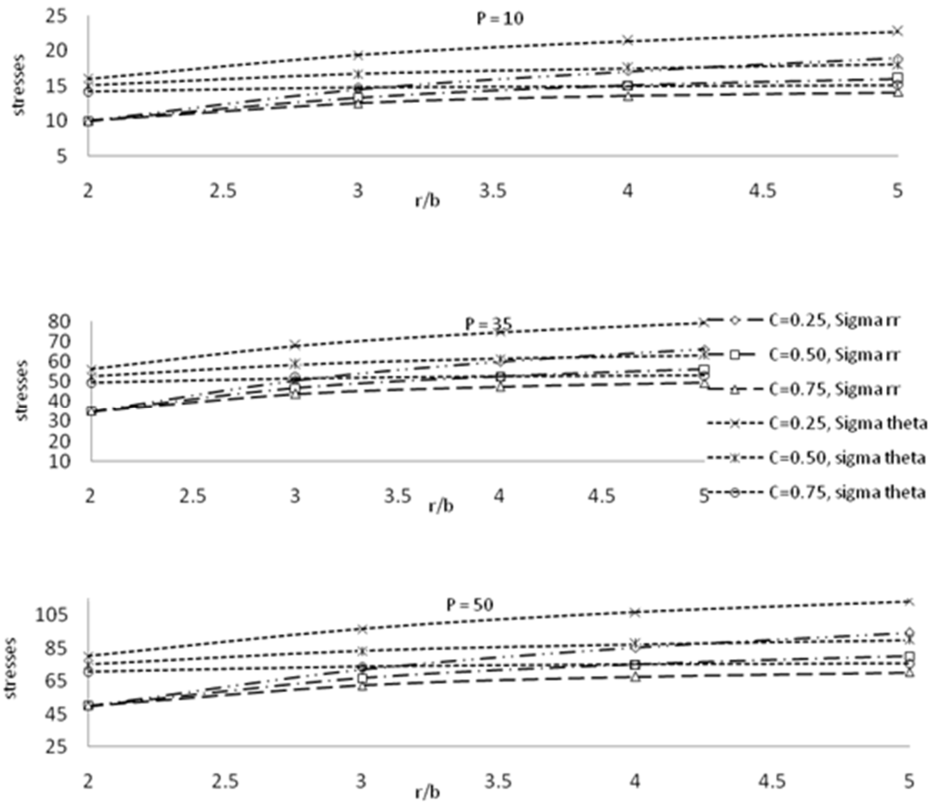

Figure3. Distribution of Elastic-plastic stresses in the spherical shell subjected to external pressure

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